Final Report

Estimating the Sediment Yield due to Bend Migration in Meandering Rivers

by

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14. ABSTRACT

Management of sediment in rivers is important to sustainable use of natural watercourses. This is addressed through Regional Sediment Management (RSM). To account for sediment inputs from bank erosion due to meander bend migration RSM needs a reliable prediction method. In this research a database was compiled on bend migration rates in the US rivers over 20 to 60 year periods. Statistical analysis was performed to establish a probabilistic relation between the bend planform and expected migration rate. Prediction of the volume of bank sediment input to the watercourse also requires estimation of the average height of the eroding bank and the length of channel along which erosion occurs. Eroding bank height and the length of eroding bankline may be estimated using an algorithm and expert judgment, respectively, based on past research performed for the USACE. The method is applicable to alluvial, meandering rivers.

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Abstract

Management of sediment in rivers is a major element of the sustainable use of natural watercourses. In the USA this is addressed through Regional Sediment Management: http://www.wes.army.mil/rsm/. To be effective, RSM needs data on sediment inputs to the fluvial system, including that from bank erosion, which may account for as much as 75% of the load. In this research a database of 1,512 meander bends on rivers in the USA was used to supply information on bend migration rates over 20 to 60 year periods. Statistical analysis was performed on the bend migration data to explore the relationship between the bend planform at the beginning of each monitoring period (represented by R/w = the ratio of bend radius to channel width) and the average rate of bend migration (represented by E/w = the ratio of the average annual retreat of the outer bank to the channel width). A probability analysis was used to derive a series of curves for E/w as a function of R/w for probabilities of the expected rate of erosion not being equaled or exceeded ranging from 10% to 99%. This analysis provides the basis for predicting the rate of bank erosion due to bend migration with a user selected probability of the rate not being equaled or exceeded. Prediction of the volume of bank sediment input to the watercourse due to bend migration also requires estimation of the average height of the eroding bank and the length of channel along which erosion occurs. The eroding bank height may be estimated using an algorithm developed in previous research performed for the USACE for average scour depth at the outer margins of meander bends (Thorne *et al.* 1995). The length of eroding bank may be estimated from field observation or using expert judgment based on consideration of the Brice data set (Soar and Thorne 1999). Methods to make these estimates are presented to enable users to predict the volume of sediment derived from bank erosion due to bend migration in single-thread, alluvial, meandering rivers.

Keywords

Bend migration   Meandering rivers   River morphology   Sediment yield

Contents

Background

Management of sediment in rivers is a major element of the sustainable development and management of natural watercourses. The strategic need to consider sediment at the catchment or regional scales emerged during the late twentieth century and led to initiatives to better represent the sediment dynamics, using approaches that are both reliable and fit the purpose. In the USA, the approach developed by the USACE is *Regional Sediment Management*:

http://www.wes.army.mil/rsm/

Within the context of Regional Sediment Management, a number of useful tools have been developed, including the Sediment Impact Assessment Method (SIAM) and a new capability to route sediment and track the morphological changes that sediment imbalances generate within HEC-RAS. While these tools offer the capability to account for sediment and its impacts at the basin or regional scale, they are reliant on good input data concerning sediment sources and rates of sediment delivery to the channel from those sources. Sources typically include not only scour of the bed but also catchment erosion, hillslope processes, gullying and bank erosion. Of these potential sources, experience from many parts of the USA suggests that bank erosion may be the main contributor of sediment - particularly that which is much finer than that found in the channel bed, and which is often termed ‘wash load’. For example, work by Andrew Simon of the USDA-ARS National Sedimentation Laboratory (personal communication 2005) has shown that bank erosion may contribute as much as 75% of the wash load in many streams and rivers in the USA.

The spatial distribution of bank erosion in meandering rivers is broadly a function of the planform pattern. In most rivers with a sinuous, meandering course bank erosion is focused on the outer bank in bends and concentrated between the bend apex and bend exit. As a result, meanders tend to grow in amplitude and migrate downstream through time – processes termed ‘extension’ and ‘migration’ respectively. It is therefore possible to estimate the bank erosion sediment yield to the stream based on knowledge of rate and pattern of meander shifting. However, shifting is an episodic process related to the magnitude and sequence of flow events competent to erode the outer banks at meander bends. It is, therefore impossible to predict in the short term as the occurrence and sequence of events is indeterminate. Conversely, over longer periods, short term fluctuations in bank erosion and meander evolution rates tend to average out and it should be possible to estimate the bank erosion sediment yield in a meandering river based on the average rate of shifting of its bends, the average height of the eroding banks and the proportion of the channel length along which the banks are retreating due to meander migration.

This hypothesis provides the basis for the research reported herein, with statistical treatment of datasets on channel planforms, meander geometries and bend shifting rates for rivers in the USA being used to provide the basis for estimating bank erosion sediment yield for sediment modelling to support improved approaches to regional sediment management.

Statement of Problem

Simply stated, the problem addressed in this research project was to derive a method of predicting the contribution of bank erosion associated with bend shifting to the sediment load in a meandering, alluvial river. This problem may be broken down into elements dealing with (1) the bend migration rate, (2) the average height of the retreating bank and (3) the length of retreating bankline within the study reach.

A further problem concerns the data required to apply the method and need for specialist expertise on the part of the individual responsible for making the prediction. For the method to have good utility, it must not have excessive data requirements and must be quick to apply. Similarly, the method must be suitable for use by engineers and river scientists with limited specialist training in fluvial geomorphology and sediment work. These requirements effectively rule out approaches based on computational fluid mechanics or morphological modelling of meandering rivers.

Approach Adopted

Overview

In light of the problem addressed and the limitations imposed by the need to produce a method that is quick to apply, requires little data and which can be used by non-specialists, an empirically-based approach was adopted for use in this research. The approach adopted may be considered in three parts, dealing respectively with predicting first, the bend migration rate, second, the average height of the retreating bank and, third, the length of retreating bankline within the study reach.

Bend Migration Rate

The database selected to provide the primary basis for the empirical approach contains information for 1,503 bends in 141 meandering reaches of 89 rivers throughout the continental USA. The database was originally assembled on behalf of the US Transportation Research Board (TRB) during research lead by Ayres and Associates and involving the University of Nottingham.

TRB's National Cooperative Highway Research Program (NCHRP) Web Document 67: Methodology for Predicting Channel Migration documents and presents the results of this study in 'NCHRP Report 533: Handbook for Predicting Stream Meander Migration', a stand-alone handbook for predicting stream meander migration using aerial photographs and maps.

A companion product to NCHRP Web Document 67 is NCHRP CD 49: Archived River Meander Bend Database, a four-CD-ROM set that contains the actual database. It is this database that provides the substantive and original basis for the statistical and probabilistic treatment reported herein.

However, the TRB database was designed for use in predicting the risk associated with meander shifting in the vicinity of bridge crossings. It was not intended for use in estimating bank erosion sediment yield due to bend shifting. Consequently, further work was required to modify the database for use in sediment yield prediction for regional sediment management application. Also, doctoral research work using the

TRB database at the University of Nottingham by Salam Sikder uncovered a considerable number of inconsistencies, errors and omitions in the TRB database. While this is to be expected in a large database of this nature, nevertheless, the database had to be quality checked and enhanced prior to its use in this new application. Statistical analysis of the quality-checked database was performed

Average Height of Retreating Bank

The approach selected to predicting the average height of the retreating bank was based on analysis of data on scour depths adjacent to the outer banks in morphologically-active meander bends that were collated as part of previous research at the University of Nottingham, performed on behalf of the USACE (Thorne et al. 1991, Thorne and Abt 1992, 1993; Thorne et al. 1995). In this study, an algorithm based on these data was chosen for prediction of the average height of retreating banks at the outer margins of migrating bends in alluvial rivers.

Length of Retreating Bankline

Bank retreat due to bend migration in a meandering river occurs along some fraction of the overall length of the bankline in a reach. Accurate prediction of the volume of material supplied to the sediment load therefore requires estimation of the proportion of the bankline that is retreating due to bend shifting. The approach adopted in this study relies on assessment of the extent to bank erosion based on either field observation or, if it is impractical to visit the study reach, expert judgment. In the case of expert judgment, reference should be made to the Brice Classification system for assessing the likely extent of bank retreat as a function of the type of meander activity displayed by the river in the study reach.

Results

Bend Migration Rate

Probabilistic Approach

Even though some bend development mechanics and bend erosion processes are known, due to complex variation in hydrologic, hydraulic, sedimentological, geotechnical settings and processes operating on a bend, it is nearly an impossible task to predict through deterministic modelling future changes of a bend, because all operating processes and interactions among them vary over time and space. Also, bank erosion and migration processes are influenced by channel change at a variety of scales: bend-scale, site-scale, reach-scale, catchment-scale.

A probabilistic approach provides a useful means of analysing the meander bend migration. This approach is purely empirical in that it is entirely based on historical data. It may appear that it does not describe the actual physical mechanics, which cause erosion, bank failure and eventually bend migration. However, migration data are historical records of the driving processes that were operating to cause bank retreat. Hence the data represent the combined outcome of all the operating processes in the bend in question. In this approach the main requirements centre on ensuring authenticity and sufficiency in the data. Here authenticity should include two things: (1) choosing appropriate variable(s) which represent the result of the past processes (2) Employing the correct method of measurement and data analsysis.

Sufficiency means the number of data points is adequate, so that required probabilistic and statistical methods can safely be employed.

The form of the predictive approach builds on the work by Hickin and Nanson. In their analysis there is a functional relationship between rate of migration and radius - both in non-dimensional form. The form of their relation has been verified by many other researchers subsequently, so a general shape of the relation is now accepted without much debate. Therefore, for to face the practical river engineering problem one can apply it in order to foresee the future magnitude of erosion by employing probabilistic analysis. This method requires historical records of changes of river course – aerial photographs and/or historical survey maps. The beauty of using this functional relation is that it requires only three parameters: 1. magnitude of migration (maximum) of bends between two successive time points (M), 2. radius of curvature at the beginning of the study period (R) and 3. river channel width (W). All three for any bend can be obtained from the aerial photographs or maps.

Taking ratio of radius of curvature to river channel width (R/W) as the independent variable and meander migration per river channel width per year (M/W/Y) as the dependent variable, one can use a bi-variate probability distribution to predict the rate of migration as a function of bend geometry for any selected probability of that rate not being equaled or exceeded. Therefore, for a given value of R/W for a bend in question we can find the probability of M/W/Y that a certain amount of migration will occur. Application of this probabilistic bend migration rate prediction tool requires only simple measurements of bend radius and channel.

This probabilistic approach is ideal given the great and unavoidable uncertainties involved in any such prediction.

Data and Variables

For the probabilistic analysis in this report we have considered the whole data set of river sites contained in the quality-screened NCHRP database held at the University of Nottingham. There are two variables involved in our probabilistic analysis: the ratio of bend radius of outer bank curvature to river channel width at the meander crossing (Rb/WbCr); to migration of the bend apex per river channel width at crossing per year (MMA/WbCr/Y). In the refined database there are 1512 bends (that means 1512 individual cases) in total. The variable Rb/WbCr is applicable to a time period between 20 and 60 years. Beginning values from each time period are used in representing bend geometry. Over time period 1, the initial value of the ratio of radius of curvature of outer bank to the initial value of river channel width at crossing (R1/W1Cr) has 1512 cases. In the same way, over time period 2, R2/W2Cr has 1512 cases. These measurements were performed completely in a independent manner, therefore, in order to make the data set bigger we combined the data on two time points together. Therefore, the date set used for probability analysis has 3024 cases.

The distribution of the data points is shown Figure 1.

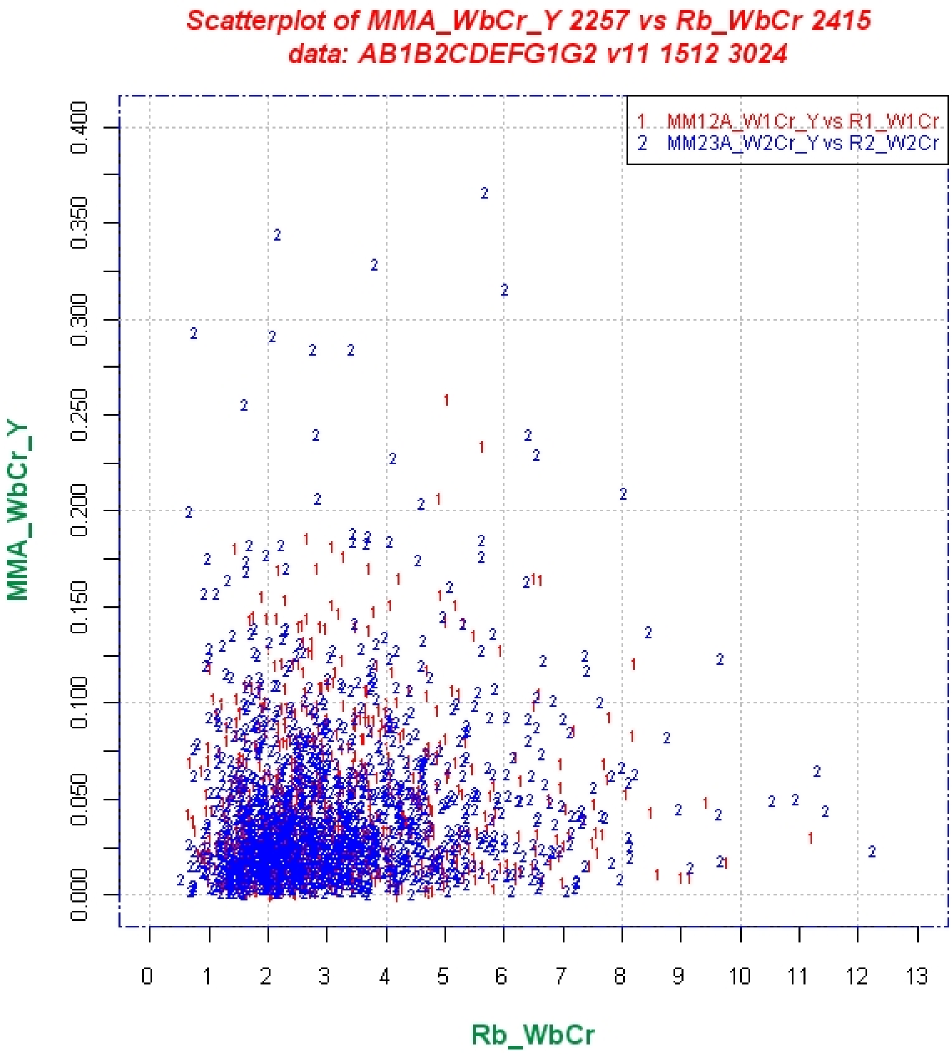

Figure 1. Scatter plot of points in the refined database. Symbols indicate whether points are from the first (1) or second (2) study period in the data set.

The figure is self explanatory; however a little clarification of the pertinent components could be worth mentioning. In this figure (in a broader sense throughout this report) the variable Rb/WbCr is used as abscissa variable and MMA/WbCr/Y as ordinate variable. In the heading the number 2415 and 2257 imply the available data points of the x and y variable, respectively. Thus it is the scatter plot composed of available data points of 2257 on both variables. The data point symbol “1” indicates the meander migration over time period 1 per river channel width at crossing per year. Likewise, the data point symbol “2” indicates the meander migration per river channel width at crossing per year over time period 2. We can see in the scatter plot that the magnitude of migration increases as ratio increases and shows a zone of maximum migration over the ranges of the ratio from 2 to 6 and then gradually decreases again.

Before the bivariate probability analysis can be performed, the uni-variate distributions of the variables should be normalized. The way that the variables were normalized is described in the next two sub-sections.

Univariate distribution and normal transformation of Rb/WbCr

The univariate distribution of the variable Rb/WbCr may be shown using the summary data in Table 1. The minimum and maximum values are 0.55 and 12.26, respectively. Hence, the maximum ratio is 22.3 times the minimum, indicating a very wide variety of bends ranging from tight elbows to long, gentle bends. The maximum data value is 12.26, but the median is only 2.7 and 3rd quartile is 3.78 - indicating a strongly right-skewed distribution.

Table 1: Summary statistics for the variable Rb/WbCr

Variable	Minimum	Lower Quartile	Median	Upper Quartile	Maximum
Rb/WbCr	0.55	1.9925	2.688	3.784	12.263
log10(Rb/WbCr)	-0.25964	0.2993983	0.4294	0.5779511	1.088597
MMA/WbCr/Y	0.00036	0.017475	0.0319	0.056996	0.366787
MMA/WbCr/Y^0.16	0.28095	0.5233434	0.5763	0.6323174	0.851738

To transform the variable Rb/WbCr so that it is normally distributed, several exploratory statistical methods were explored, including: normal probability plot, empirical distribution function plot, histogram plot and box plot. As mathematical means both power and logarithmic transformation techniques were tested. Over a huge range of powers and bases a trial-and-error approach used for each exploratory techniques with a view to arriving at a conclusion that fitting the Gaussian model to the data is satisfactory. In this attempt of fitting a visual judgment along with numerical output was applied in the justification towards normality. For this variable a common logarithmic transformation does satisfactorily follow a normal distribution according to the Shapiro-Wilk normality test (p-value for untransformed variable (2.2*10-16), compared to 0.002726 for the transformed variable).

A single test would not usually be sufficient to support the assumption of normality. However, one should recall the fact that in case of this test the sample is very large and so even very small departures from normality would lead to rejection of null hypothesis. Therefore, in fine we can safely and satisfactorily conclude that the transformed variable $\log_{10}$(Rb/WbCr) does follow a normal distribution.

Univariate distribution transformation of MMA/WbCr/Y

Exactly the same statistical methods, techniques (both exploratory and inferential) and sequence of attempts and procedures were employed in investigating the distribution of MMA/WbCr/Y and its transformation into a normally distributed variable. Tukey’s five number summary showed that MMA/WbCr/Y also has a right skewed distribution and does not follow a normal distribution. It is worth mentioning here that this variable could be transformed into a normal variable using a logarithmic transformation. However, a power-transformed variable $(\mathrm{MMA/WbCr/Y})^{0.16}$ does follow a normal distribution.

Bivariate Analysis: Conditional Probability of MMA/WbCr/Y given Rb/WbCr

A bivariate, probabilistic analysis was performed on the normalised variables. The joint densities of the two variables were computed by employing two dimensional kernel density estimation.

Analytical Tool for Probabilistic Prediction of Bend Migration rate

Using fitted normal conditional density function as described above, the rate of meander migration for a bend with a given radius to width ratio has been obtained at selected probability levels ranging from 10% to 99% probability of the predicted rate not being equaled or exceeded (Figure 2).

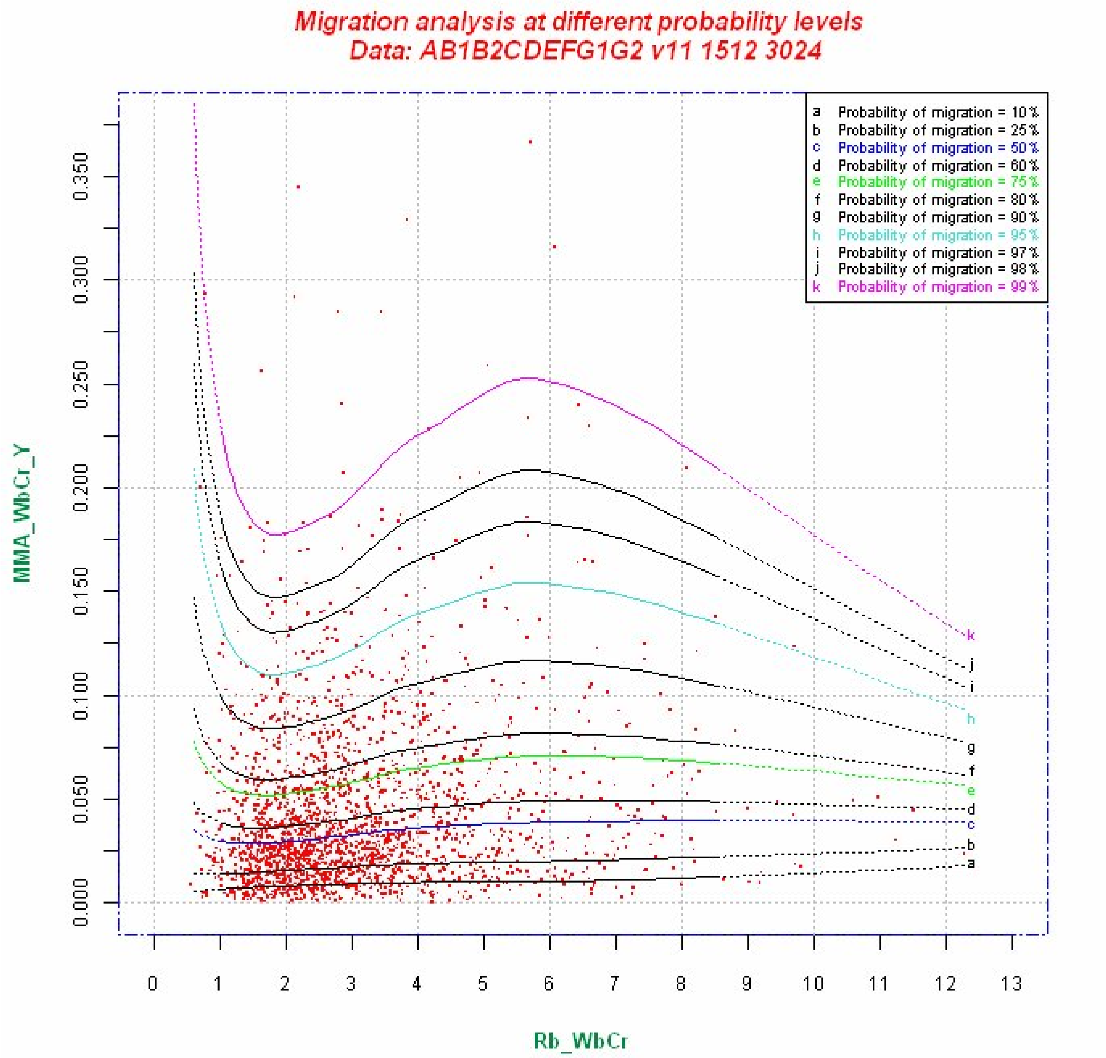

Figure 2. Probablistic analysis of bend migration rate as a function of bend geometry

Figure 2 shows three zones in probability curves. The left zone - where the curves are represented by dotted lines covers the range of the graph where values in the variable Rb/WbCr are less than 1. The middle zone, where the curves are represented by solid lines is in the range of values 1 < Rb/WbCr < 8.5. The right zone, where the curves are again represented by dotted lines, is for Rb/WbCr values greater than 8.5. In the left and the right zones, predicted migration rates are lower than in the middle zones, but the conditional densities may be affected by the sparcity of data. Hence, less confidence may be placed in the predictive curves and so they are shown as dotted lines. In assessing probable erosion rate the person performing the prediction should be careful about making predictions for bends in these zones or might better choose not to attempt a prediction when ratio of bend radius to width is either less than unity or more than 8.5.

Average Height of Retreating Bank

Length of Retreating Bankline

Estimation of the proportion of bankline length within a study reach that is actively retreating due to bend migration is best based upon field reconnaissance to identify the extent of bank erosion. A suitable approach to performing field reconnaissance of eroding banks was developed for the USACE through at the University of Nottingham (Thorne 1993) and later published in a field guide (Thorne 1998).

However, if it is not feasible to perform a field reconnaissance, then a first approximation of the proportion of bankline that is likely to be retreating due to bend migration may be made on the basis of examination of a suitable air photograph and classification of the type of meandering exhibited in the study reach according to the Brice system. For an account of the Brice approach and the database upon which it is based, see the USACE report by Soar and Thorne (2001).

The Brice classification considers the planform appearance and morphology of the channel as it appears in an air photograph to assign the stream to one of seven classes of meandering (Figure Y). Generally, the extent of lateral morphological activity and, therefore, the proportion of the bankline that is retreating due to bend migration, increases from Types A to F in the Brice classification. The proportion of bankline retreating in Type G meandering is similar to that in Type B channels.

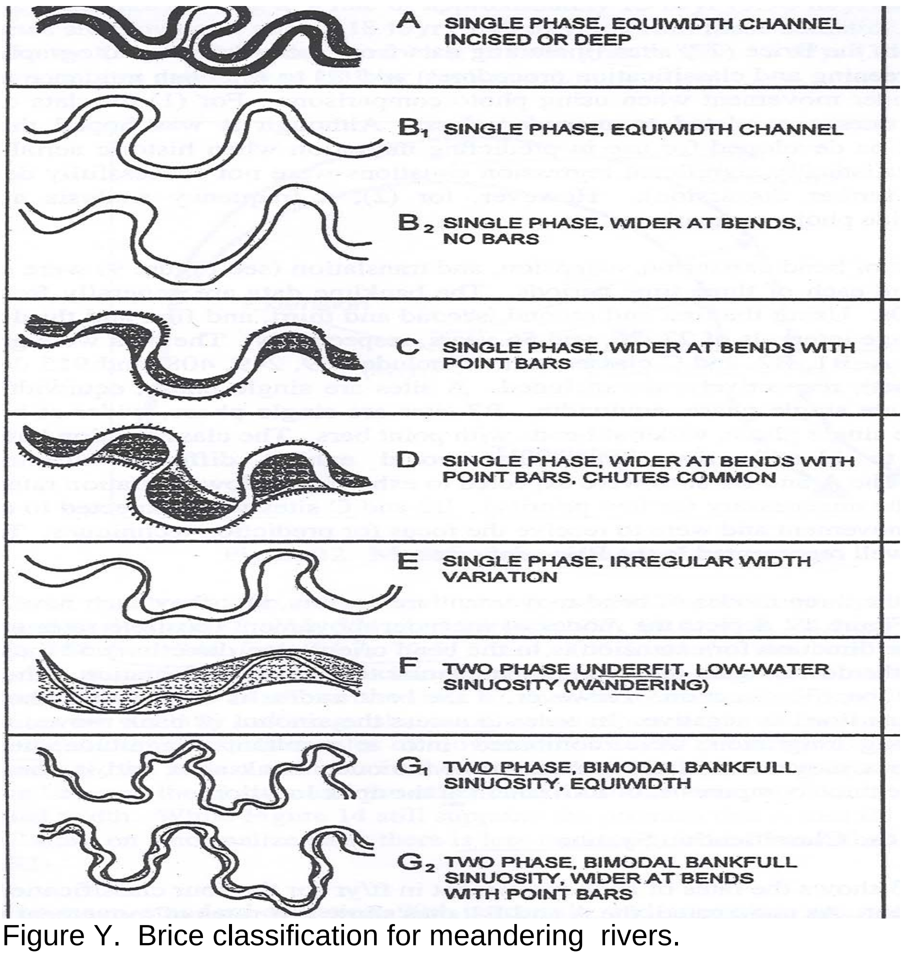

Figure Y. Brice classification for meandering rivers.

On the basis of experience and expert judgment, a first approximation of the proportion of the bankline that may be expected to be retreating is suggested in Table X. It should be stressed that the estimates produced using this approach are only first approximations and are subject to considerable uncertainty. To reflect this, Table X includes upper and lower estimates for the proportion of eroding bankline as well as the best estimate. When calculating the yield of bank sediment due to bend migration, it is recommended that the sensitivity of the calculation due to uncertainty is accounted for using upper and lower bound estimates as well as the best estimate.

Table X. First approximation approach to estimating the proportion of the bankline that is retreating due to bend migration in a meandering stream, as a function of the Brice Class.

	Proportion of bankline eroding due to bend migration		
Brice Class	**Best estimate**	**Lower estimate**	**Upper estimate**
A	0.1	0.05	0.15
B1 or G1	0.1	0.05	0.15
B2 or G2	0.2	0.15	0.25
C	0.25	0.2	0.3
D	0.3	0.2	0.4
E	0.4	0.3	0.5
F	0.5	0.4	0.6

Estimating the bank erosion sediment yield

The annual yield of sediment due to bank erosion may be estimated by multiplying together the bank retreat rate, average bend scour depth and length of the bank line that is retreating due to bend migration within the study reach. The relevant equation is:

$BESY = BMR \times S_d \times E_{bl}$

Where, BESY = Bank Erosion Sediment Yield (M3/yr), BMR = Bend Migration Bend (m/y) for the selected probability of being not equalled or exceeded, S_d = average bend scour depth (m) and E_{bl} = length of eroding bankline in study reach (m), which is given by:

$E_{bl} = P_{bl} \times Bl$

Where, P_{bl} = proportion of bankline that is eroding due to meander migration and Bl = total length of bankline in study reach.

Discussion

The method presented in this report represents a rapid and relatively simple solution to the problem of estimating the bank erosion sediment yield in meandering rivers for inclusion in Regional Sediment Management applications.

The method is based on doctoral research performed as part of studies in fluvial geomorphology performed by Salam Sikder under the supervision of Colin Thorne. Consequently, the approach should be regarded as preliminary and subject to further and on-going development.

The method is based on a large data sets and experience gained in studying alluvial streams over many years and so it should be reasonably robust. However, estimation of the bank erosion sediment yield is subject to high uncertainty and, inevitably, the error margins on estimates made using this approach are likely to be high, especially if stream reconnaissance to obtain ground truth on the extent of bank erosion in the study reach is not feasible.

Uncertainty may be managed by selecting the appropriate level of probability of the estimated bank erosion rate not being equalled or exceeded. If it is important to avoid under-estimating the bank erosion sediment yield, the upper quartile (p = 75%) of the probability distribution should be selected. If the best estimate is required, the median erosion rate (p = 50%) should be used. If it is important that the bank erosion sediment yield is not over estimated, the lower quartile (p = 25%) probability level might be selected.

Conclusion and Recommendation

It may be concluded that the method presented in this report should be useful in the context of Regional Sediment Management and the application of sediment budgeting models such as SIAM.

It is strongly recommended that the method be tested and trialled by the USACE prior to its application to applied studies in order to validate and calibrate the approach.

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